Zero-Value and Correlation Attacks on CSIDH and SIKE†

Institute of Information Science - Academia Sinica

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Who am I?

- Fabio Campos, https://sopmac.org
- since 2018
  - external PhD student at Radboud University, NL
  - supervisor: Peter Schwabe
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- born in Rio de Janeiro, BR (no questions about football) ;-(

Wait, what?

Zero-Value and Correlation Attacks on CSIDH and SIKE

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Zero-Value and Correlation Attacks on CSIDH and SIKE

- side-channel attacks
- power analysis whether secret computation pass over certain values
- leaking secret information

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Zero-Value and Correlation Attacks on CSIDH and SIKE

- isogeny-based schemes
- CSIDH : allows for non-interactive key exchange
- SIKE : key encapsulation mechanism

Agenda

- Zero-value attacks
- Isogeny paths in CSIDH
- Vulnerable curves in CSIDH
- Attacking SQALE and CTIDH
- Simulation
- Countermeasures
- Applicability to SIKE

Zero-value attacks

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- Does this work in CSIDH too?

Isogeny paths in CSIDH

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- Prime of the form $p = 4 \cdot \ell_1 \cdot \dots \cdot \ell_n - 1$ with small distinct odd primes ℓ_i.

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- CSIDH isogeny graph: union of these cycles

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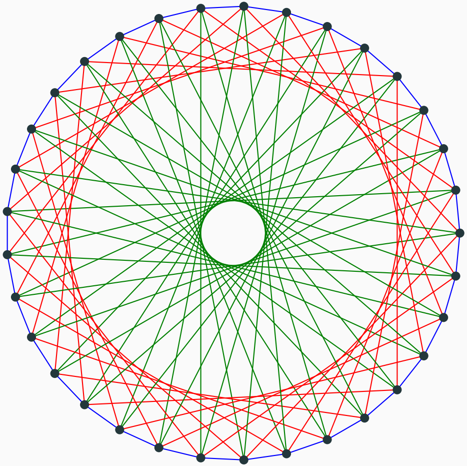
Toy example: $p = 659 = 4 \cdot 3 \cdot 5 \cdot 11 - 1$

Alice

secret: $(+, +, -)$



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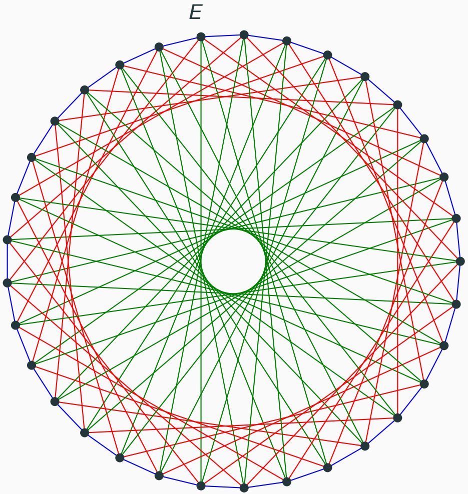

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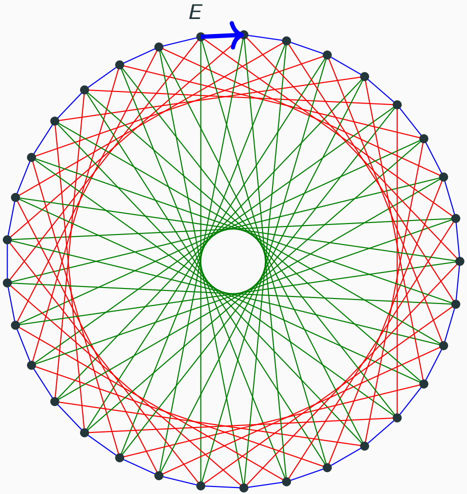

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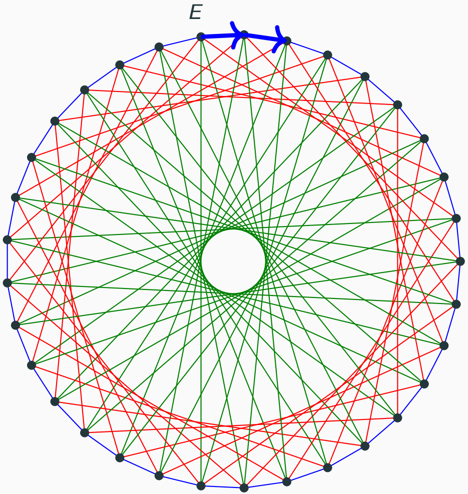

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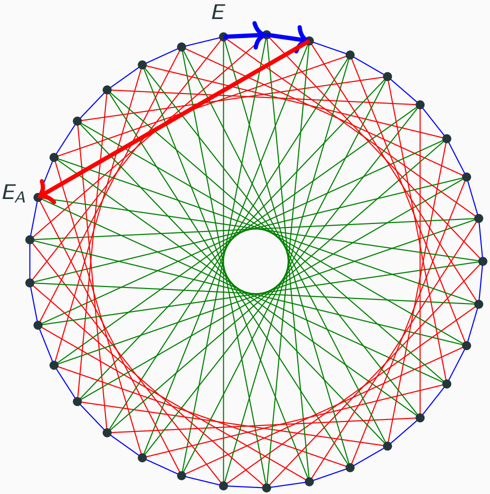

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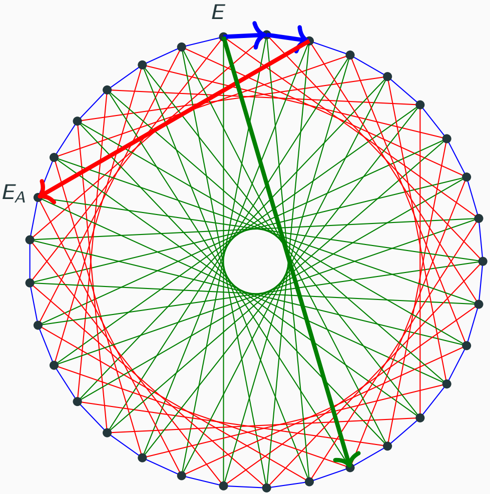

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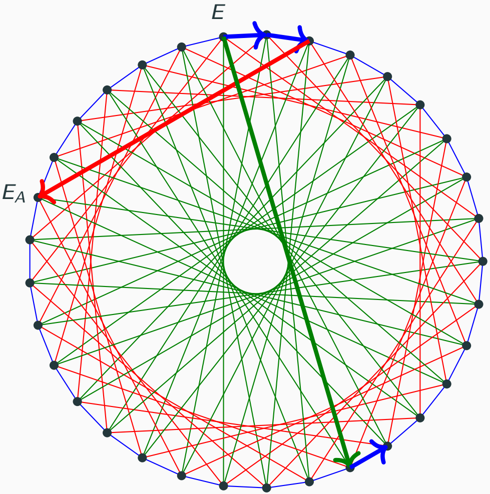

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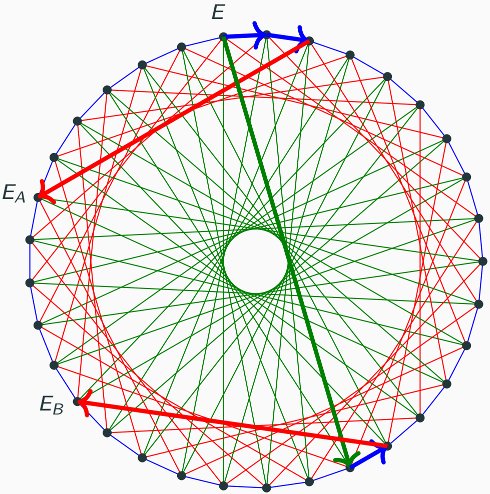

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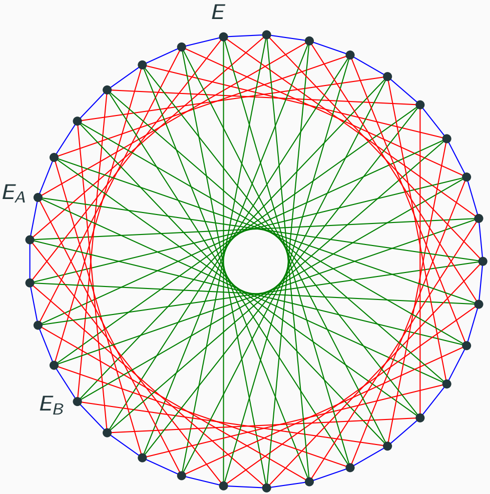

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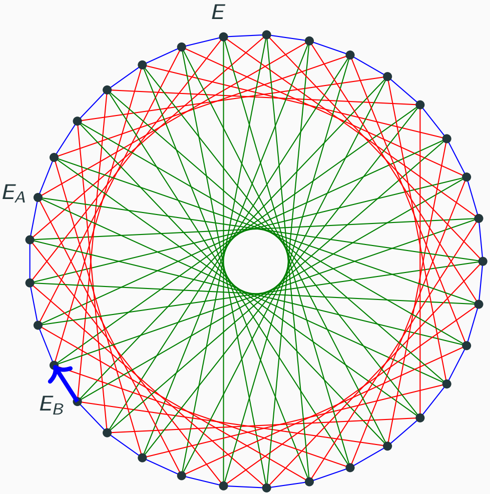

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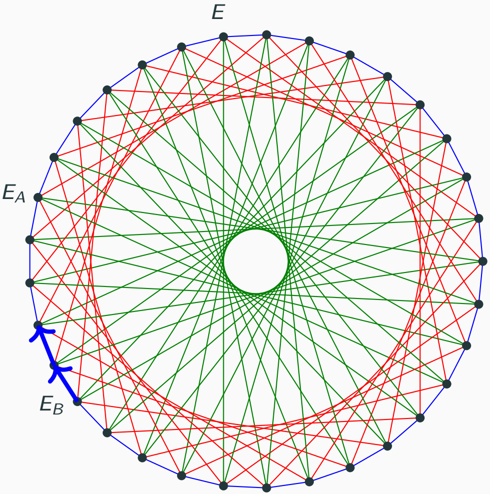

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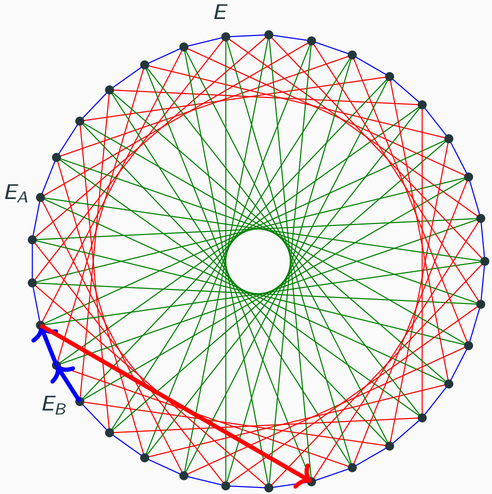

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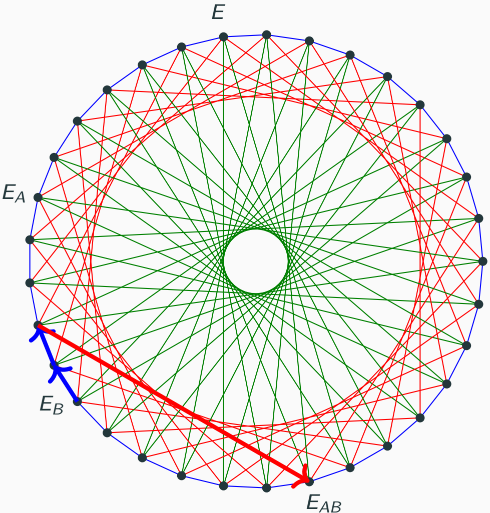

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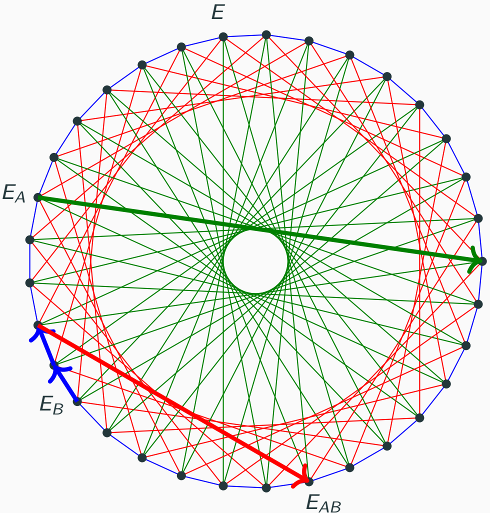

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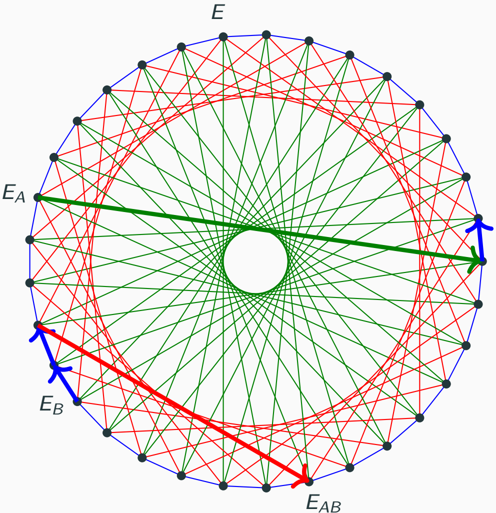

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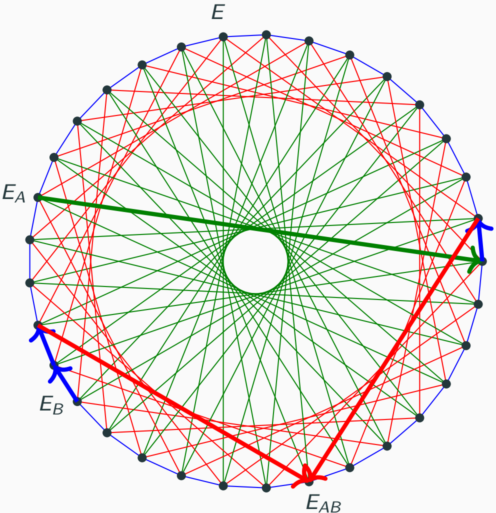

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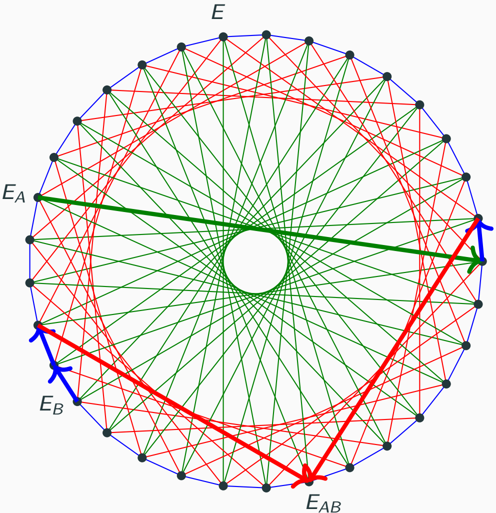

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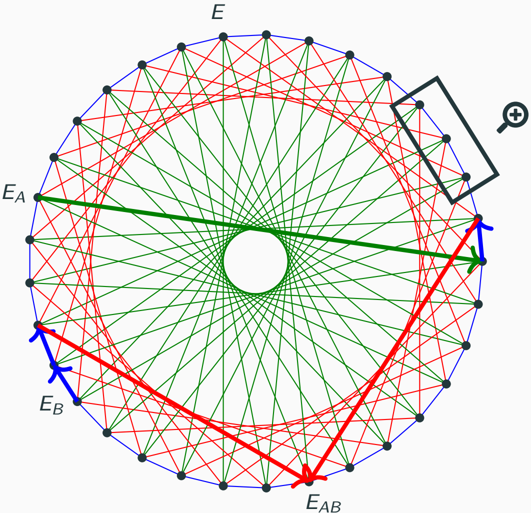

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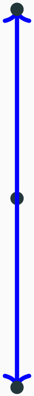

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- Montgomery form $(A : C)$ with $a = A/C$ and C non-zero
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- Strongly-correlated representation: Represents the Montgomery coefficient $a \in \mathbb{F}_p$ in projective coordinates $(\alpha : \beta)$ such that the bit representations of α and β are bit shifts.

Vulnerable curves in CSIDH

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Also works for E_6 in alternative Montgomery form: $(8C : 4C)$ with $C \in \mathbb{F}_p$ is strongly-correlated if $4C < p/2$.

Attacking SQALE and CTIDH

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- Constant-time CSIDH usually has an ordered evaluation of isogenies (modulo point rejections).
- Task: Find out the direction of the next step (also considering dummy isogenies).

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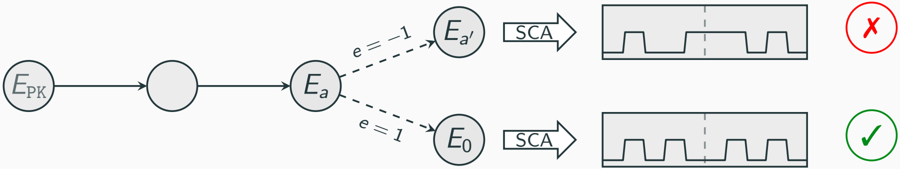

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- Ordered evaluation
- Adaptively recover key bits e_i with the generic approach.
- Each step can fail with a probability of $1/\ell_i$
  $\rightsquigarrow$ increases the number of measurements.

CTIDH [Banegas–Bernstein–Campos–Chou–Lange–Meyer–Smith–Sotáková-2021]

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Figure 1: CTIDH aka *the isogeny bus*[1]

[1]Talk by Krijn Reijnders: `https://tinyurl.com/CTIDHBeepBeep`
Original pic of bus by Teddy O on `https://unsplash.com/photos/jtpcrnqP2Mc`

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- CTIDH uses a more complicated key space and hides the actual isogeny degrees in use.
- CTIDH uses an ordered evaluation, but we have to guess the direction and the degree of each isogeny.

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CTIDH [Banegas–Bernstein–Campos–Chou–Lange–Meyer–Smith–Sotáková-2021]

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  $\rightsquigarrow$ we can detect E_0.
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- CTIDH uses an ordered evaluation, but we have to guess the direction and the degree of each isogeny.
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- This increases the number of measurements.

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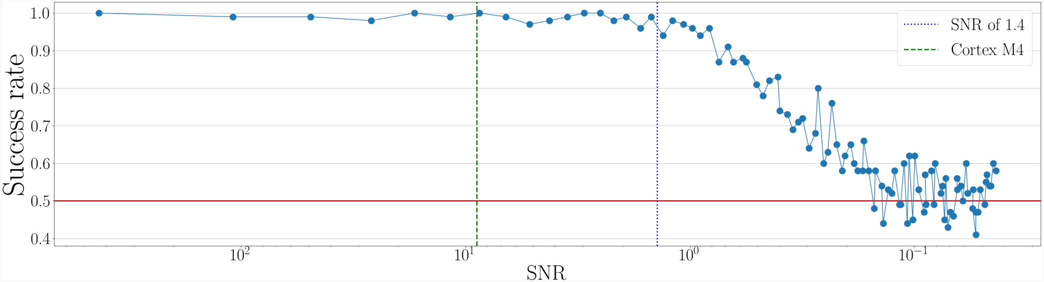
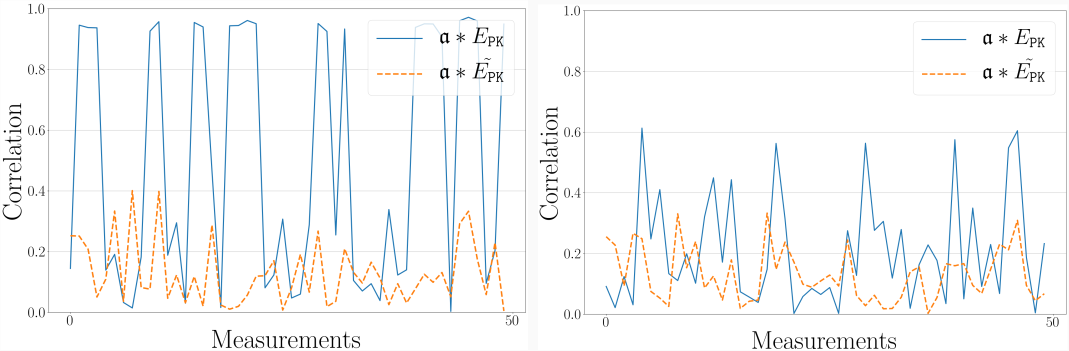
- average #measurements in SQALE-2048: 8,273
- average #measurements in CTIDH-511: 85,000

Simulation

Simulation for different noise levels (signal-to-noise-ratio):



Simulation



(a) Correlation results without noise. **(b)** Correlation results with SNR of 1.40.

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- Masking isogeny: Compute $\mathfrak{a} * E$ as $\mathfrak{z}^{-1} * (\mathfrak{a} * (\mathfrak{z} * E))$ with a masking isogeny $\mathfrak{z} * E$ of key space 2^k.
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[1] Thanks to the reviewers for this suggestion!

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- Move to the surface:[1] Pick $p \equiv 7 \mod 8$ and work on the surface of the isogeny graph (see [Castryck-Decru-2020]).
  $\rightsquigarrow$ We are not aware of vulnerable curves in this setting.

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Applicability to SIKE†

SIKE

- The attack applies to SIKE too: E_0 and E_6 are valid curves in SIKE

Figure 2: Who is next?[1]

[1] PQC^2 : Post-Quantum Cryptography Cemetery
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SIKE

- The attack applies to SIKE too: E_0 and E_6 are valid curves in SIKE
- Attack guesses secret bits/trits and detects which leads to path over E_0 or E_6.
- Required number of samples:

Scheme	SIKEp434	SIKEp503	SIKEp610	SIKEp751
Samples	228	265	320	398

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Zero-Value and Correlation Attacks on CSIDH and SIKE†

Thank you!

Paper: https://eprint.iacr.org/2022/904.pdf

Simulation: https://github.com/PaZeZeVaAt/simulation