Patient Zero & Patient Six: Zero-Value and Correlation Attacks on CSIDH and SIKE†

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Outline

- Zero-value attacks
- Isogeny paths in CSIDH
- Vulnerable curves in CSIDH
- Attacking SQALE and CTIDH
- Countermeasures
- Applicability to SIKE

Motivation: Zero-value attacks

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- Does this work in CSIDH too?

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- Prime of the form $p = 4 \cdot \ell_1 \cdot \cdots \cdot \ell_n - 1$ with small distinct odd primes ℓ_i.

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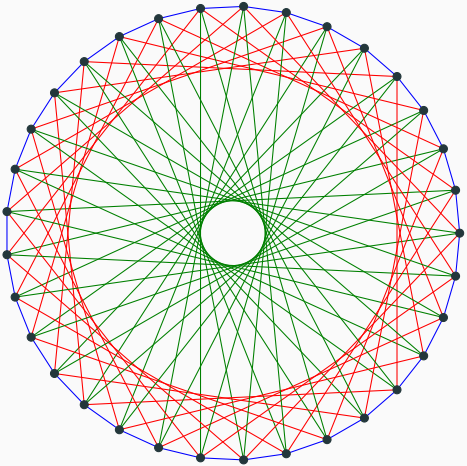
- CSIDH isogeny graph: union of these cycles

CSIDH

Toy example: $p = 659 = 4 \cdot 3 \cdot 5 \cdot 11 - 1$

Alice

secret: $(+, +, -)$



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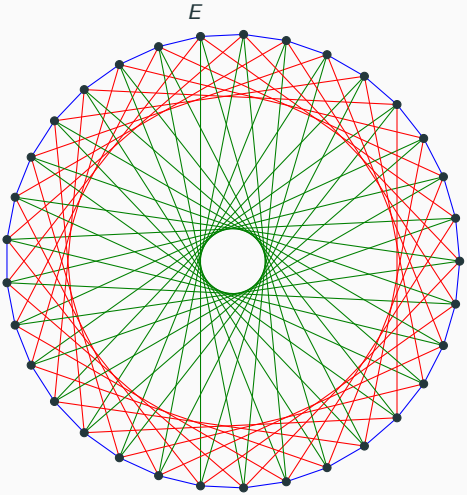

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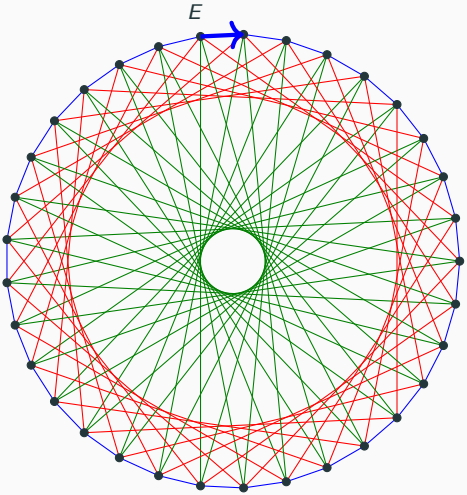

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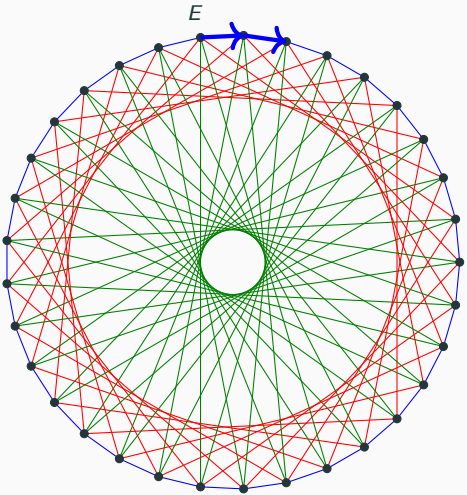

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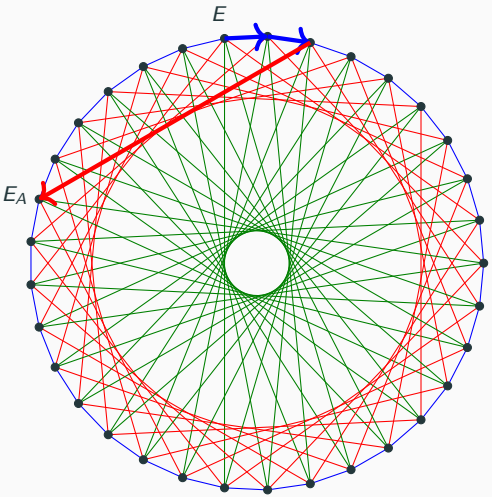

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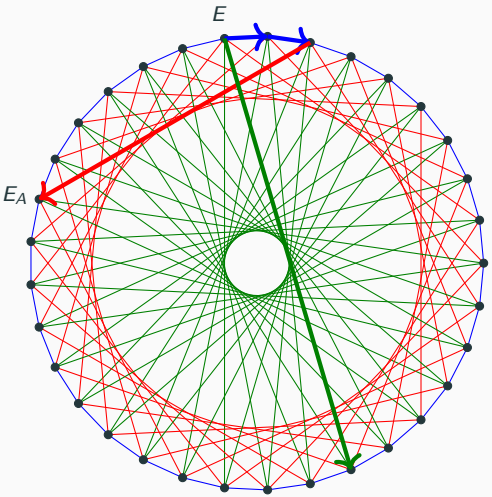

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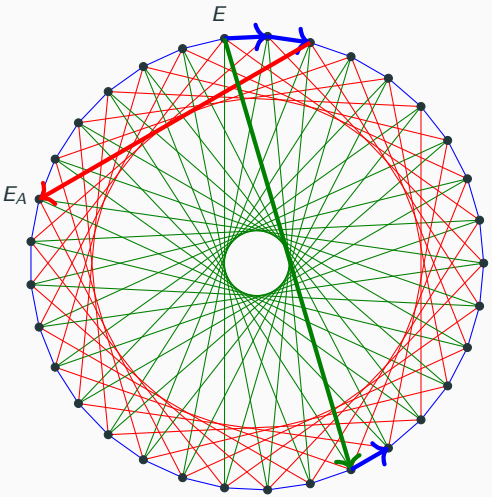

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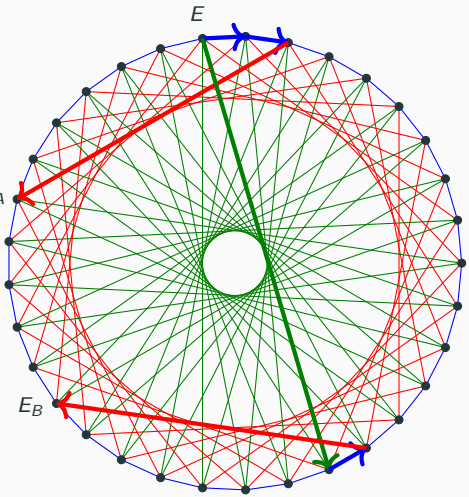

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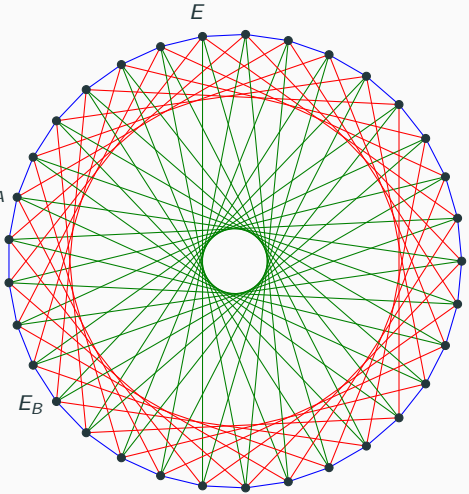

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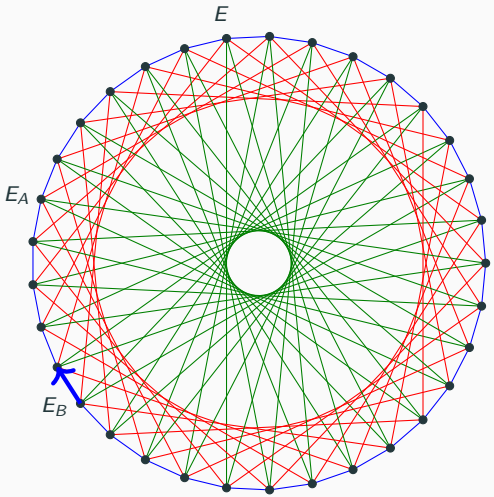

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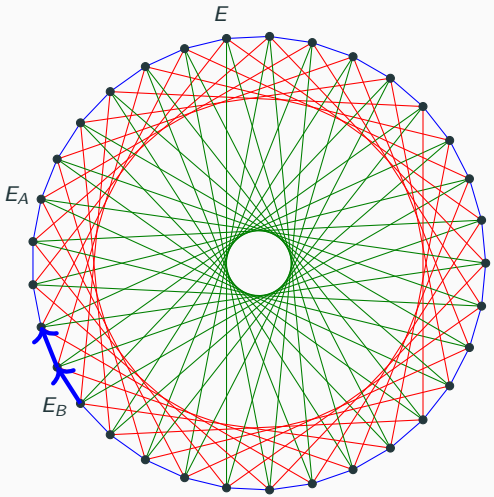

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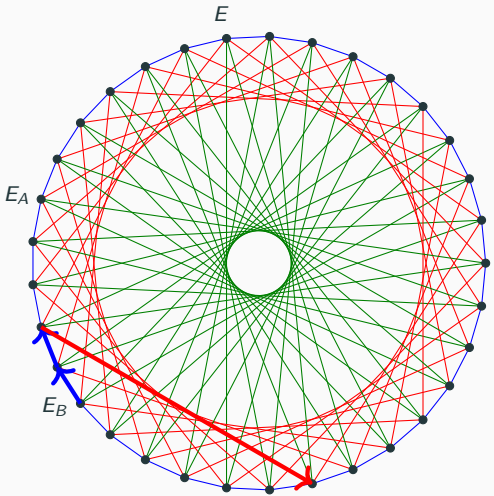

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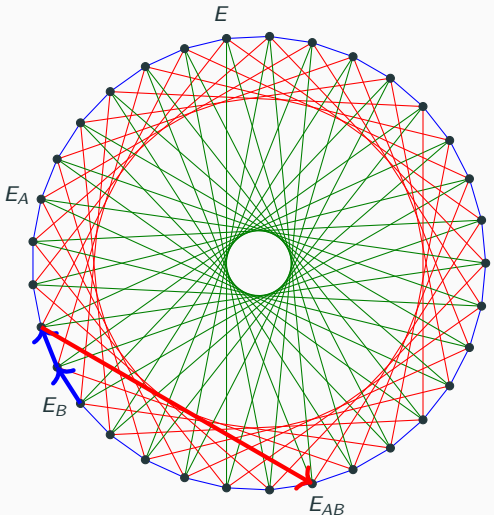

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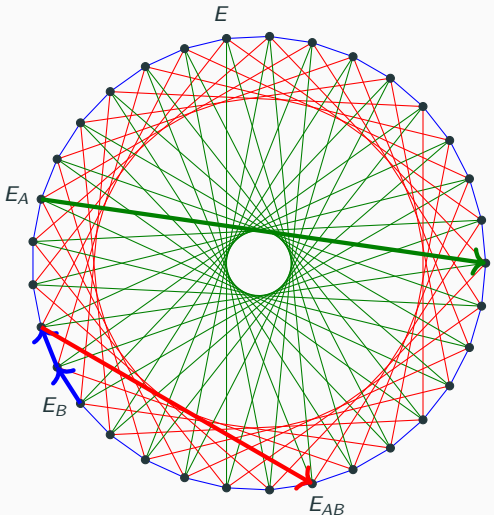

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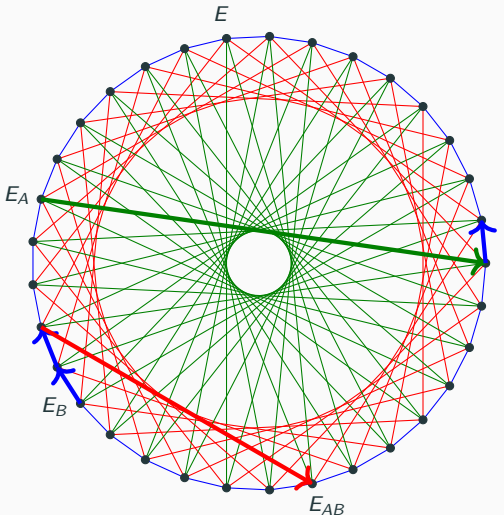

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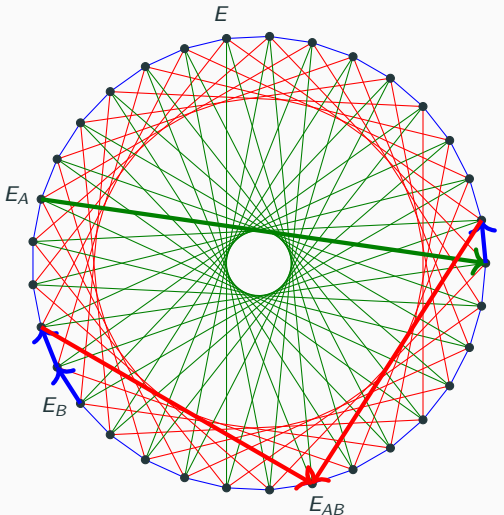

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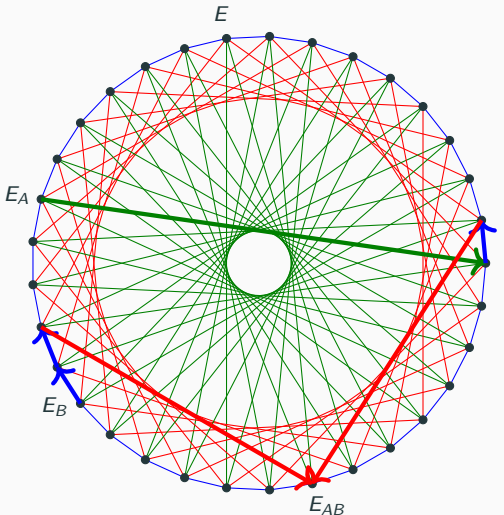

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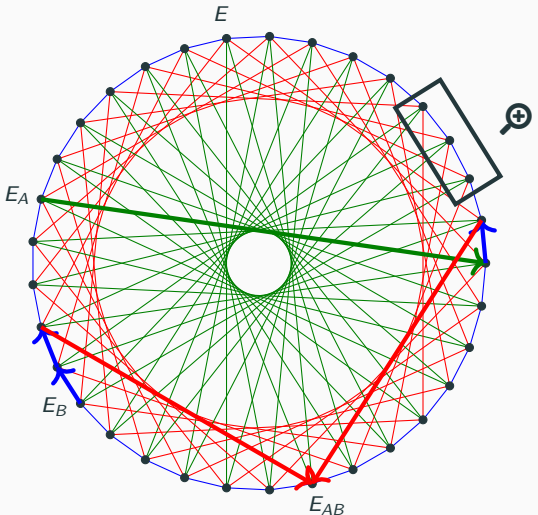

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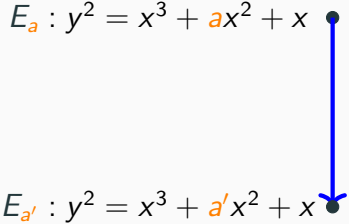

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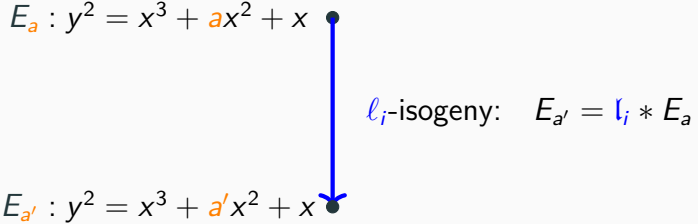

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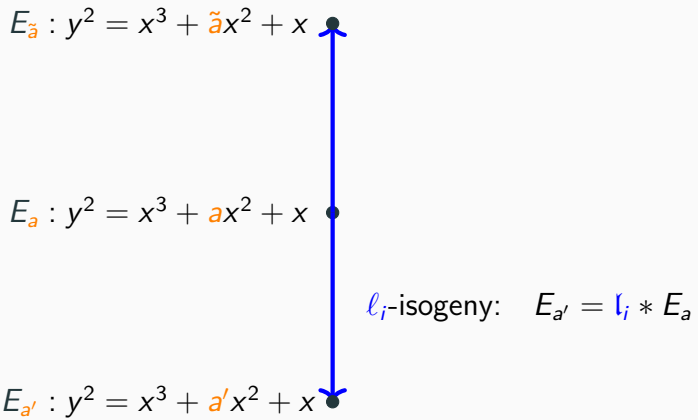
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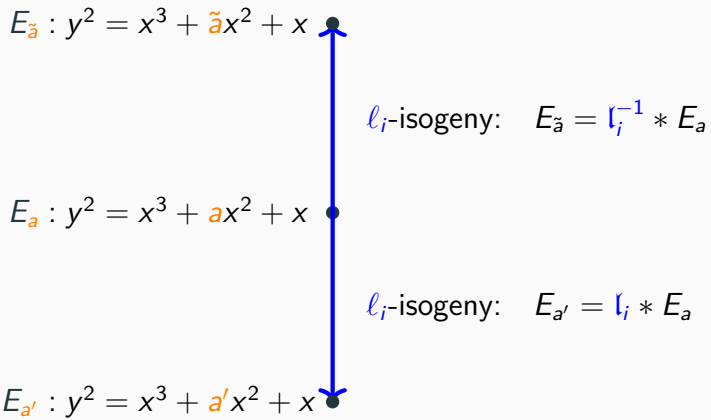
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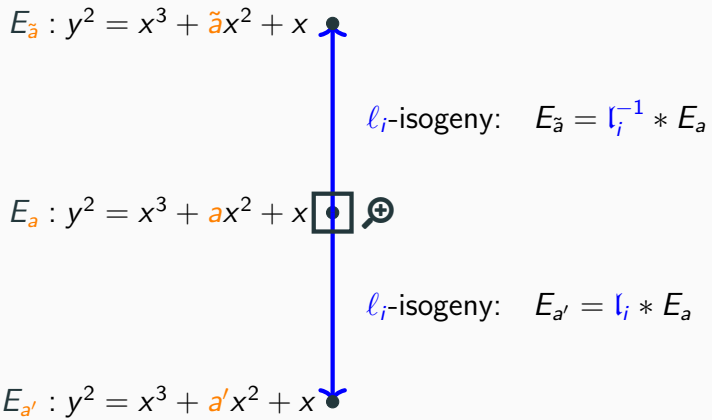
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- Zero-value representation: Represents the Montgomery coefficient $a \in \mathbb{F}_p$ in projective coordinates $(\alpha : \beta)$ such that $\alpha = 0$ or $\beta = 0$.
- Strongly-correlated representation: Represents the Montgomery coefficient $a \in \mathbb{F}_p$ in projective coordinates $(\alpha : \beta)$ such that the bit representations of α and β are bit shifts.

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Also works for E_6 in alternative Montgomery form: $(8C : 4C)$ with $C \in \mathbb{F}_p$ is strongly-correlated if $4C < p/2$.

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- Constant-time CSIDH usually has an ordered evaluation of isogenies: $\mathfrak{l}^{(n)} * \cdots * \mathfrak{l}^{(1)} * E$ (modulo point rejections).
- Task: Find out if $\mathfrak{l}^{(k)} = \mathfrak{l}_i^e$ for $e = 1$ or $e = -1$ (or $e = 0$ if the implementation uses dummy isogenies).

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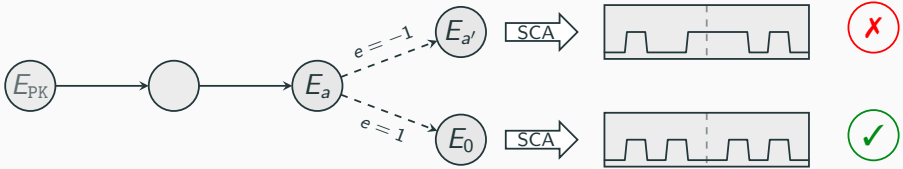

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Simulation of our attacks exploiting strong correlation:

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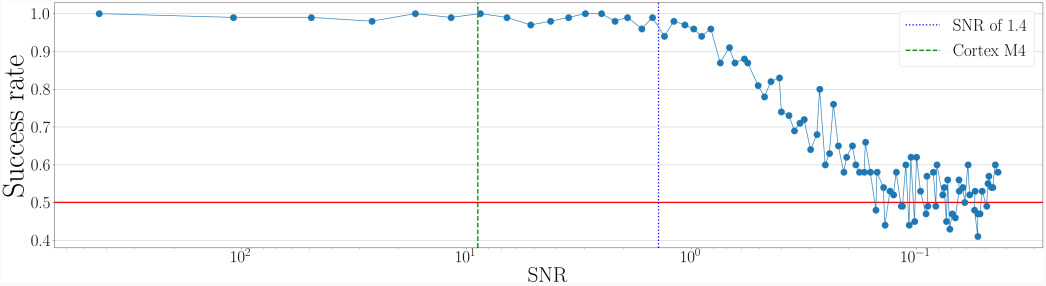
- average #measurements in CTIDH-511: 85,000

Simulation

Simulation for different noise levels (signal-to-noise-ratio):



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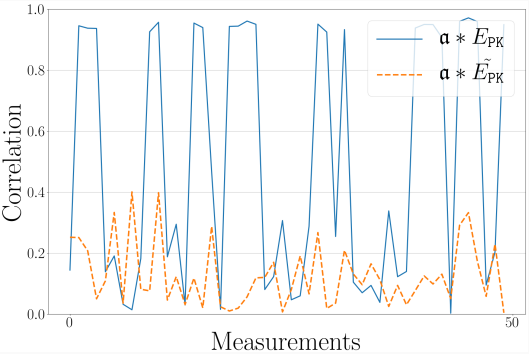
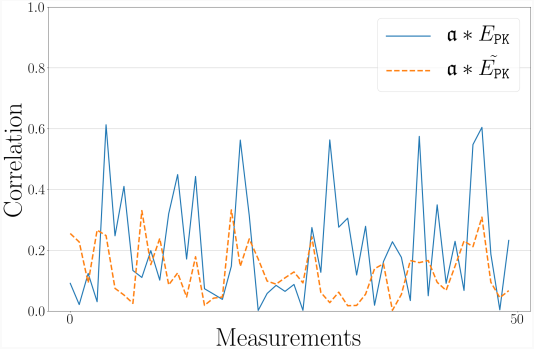


(a) Correlation results without noise.



(b) Correlation results with SNR of 1.40.

Countermeasures

- Masking isogeny: Compute $\mathfrak{a} * E$ as $\mathfrak{z}^{-1} * (\mathfrak{a} * (\mathfrak{z} * E))$ with a masking isogeny $\mathfrak{z} * E$ of key space 2^k.
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- Move to the surface:[1] Pick $p \equiv 7 \mod 8$ and work on the surface of the isogeny graph (see [Castryck-Decru-2020]).
  $\rightsquigarrow$ We are not aware of vulnerable curves in this setting.

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- Required number of samples:

Scheme	SIKEp434	SIKEp503	SIKEp610	SIKEp751
Samples	228	265	320	398

Patient Zero & Patient Six: Zero-Value and Correlation Attacks on CSIDH and SIKE†

Thank you!

Paper: https://eprint.iacr.org/2022/904.pdf

Simulation: https://github.com/PaZeZeVaAt/simulation