Code-based cryptography: 101

Class 2

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June 1, 2022

Outline

McEliece vs Niederreiter

Signatures

Code-based cryptography 101

Recap of McEliece...

- Let C be a length-n binary Goppa code Γ of dimension k with minimum distance $2t+1$ where $t \approx (n-k)/\log_2(n)$; original parameters (1978) $n = 1024$, $k = 524$, $t = 50$.
- The McEliece secret key consists of a generator matrix G for Γ, an efficient t-error correcting decoding algorithm for Γ; an $n \times n$ permutation matrix P and a nonsingular $k \times k$ matrix S.
- n, k, t are public; but Γ. P, S are randomly generated secrets.
- The McEliece public key is $k \times n$ matrix $G' = SGP$.

Niederreiter cryptosystem

Niederreiter

- Use $n \times n$ permutation matrix P and $(n-k) \times (n-k)$ invertible matrix S.
- Generate the parity matrix H, for a linear code (usually Binary Goppa Code). The public key is $K = SHP$. The private key is (S, H, P)

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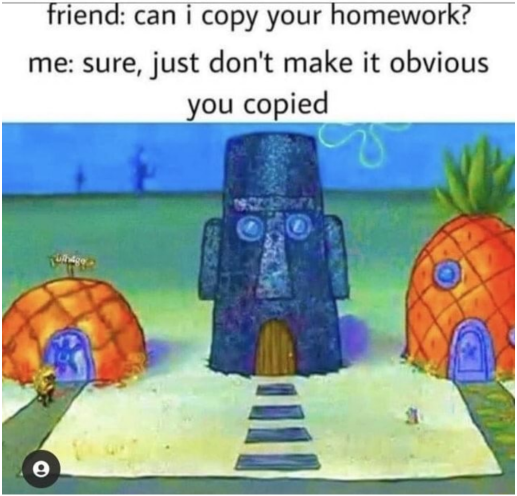

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Originally, Niederreiter proposed in 1986 the scheme with Reed-Solomon codes. However, it was broken in 1992[1].

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Attacks against code-based

Attacks!

- Information Set Decoding: [Prange, 62]
- Relax the weight profile: [Lee & Brickell, 88]
- Compute sums on partial columns first: [Leon, 88]
- Use the birthday attack: [Stern, 89], [Dumer, 91]
- First "real" implementation: [Canteaut & Chabaud, 98]
- Initial McEliece parameters broken: [Bernstein, Lange, & Peters, 08]
- Lower bounds: [Finiasz & Sendrier, 09]

Attacks against code-based

Attacks!!

- Asymptotic exponent improved [May, Meurer, & Thomae, 11]
- Decoding one out of many [Sendrier, 11]
- Even better asymptotic exponent [Becker, Joux, May, & Meurer, 12]
- "Nearest Neighbor" variant [May & Ozerov, 15]
- Sublinear error weight [Canto Torres & Sendrier, 16]
- McEliece needs a Break – Solving McEliece-1284 and Quasi-Cyclic-2918 with Modern ISD [Esser, May, & Zweydinger, 21]

McEliece vs Niederreiter cryptosystem

Niederreiter

- McEliece:
  - Created in 1978;
  - It uses Binary Goppa Codes;
  - **Public Key:** $(k \times n)$ matrix $G' = SGP$;
  - **Private Key:** Γ, P, S.
- Niederreiter:
  - Created in 1986;
  - Originally, it uses Generalized Reed-Solomon codes (but it was broken)
  - For security, it uses Binary Goppa Codes;
  - **Public Key:** $((n - k) \times n)$ matrix $H' = SHP$;
  - **Private Key:** H, P, S.

SPOILER ALERT!

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This is the scenario in code-based signatures:

Digital Signatures

Digital Signatures

The main idea in a signature scheme is:

- Take the hash of a message m, such as $h = H(m)$;
- Sign h with a private key sk;
- Publish h and pk. Anyway can verify that m was properly signed, and it is valid.

Digital Signatures

Digital Signatures

The main idea of a Hash-and-Sign scheme is:

- Take the hash of a message m, such as $h = H(m)$;
- Sign h with a private key sk;
- Publish h and pk. Anyway can verify that m was properly signed, and it is valid.

Digital Signatures

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How it is possible to do it in Code-based?
Let $H \in \mathbb{F}_2^{r \times n}$ a parity check matrix of a t-error correcting Goppa code.

Digital Signatures

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How it is possible to do it in Code-based?
Let $H \in \mathbb{F}_2^{r \times n}$ a parity check matrix of a t-error correcting Goppa code. Signing:

- Hash the message m into $h(m) = s \in \mathbb{F}_2^r$;
- Find e of minimal weight such that $eH^T = s$;
- Use e as a signature.

Verification:

- hash the message m into $h(m) = s \in \mathbb{F}_2^r$;
- verify if $eH^T \stackrel{?}{=} s$.

CFS Signatures

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In 2001, N. Courtois, Finiasz and Sendrier (CFS) published "How to achieve a McEliece-based digital signature scheme".[2] The parameters were:
$n = 2^m = 2^{16}$, $t = 9$, $r = n - k = tm = 144$.

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** All this was using binary Goppa Codes **.

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Besides the Goppa codes, there is Low Rank Parity Check (LRPC) codes. It doesn't use the "Hamming" metric, it uses rank metric. In 2017, RankSign was published as a signature[4].

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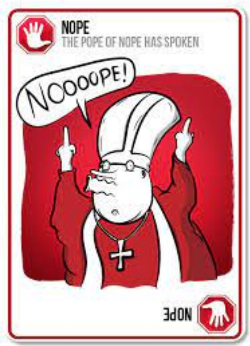
In 2018, it was broken. [a]

[a] T. Debris-Alazard and J.-P. Tillich. Two attacks on rank metric code-based schemes: RankSign and an IBE scheme. In International Conference on the Theory and Application of Cryptology and Information Security, pages 62–92. Springer, 2018.



Signature in Code-based

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All broken signatures in code-based:
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All broken signatures in code-based:
CFS, RankSign, RaCCos, pqsigRM, LXY, KKS, and goes on...
'Safe' signature (so far): Wave, Durandal, and LESS.

Wave Code-based

Wave signature

Wave signature is a hash-and-sign. It was presented in 2019 at Asiacrypt.
The Wave trapdoor is built from two random linear codes U and V of length $n/2$ and dimensions k_U and k_V, respectively, over $\mathbb{F}_q$.
The codes U and V are combined to form a code W of length n, and dimension $k = k_U + k_V$.

Wave Code-based

Wave signature

The public key is a parity-check matrix $H \in \mathbb{F}_q^{(n-k \times n)}$ for the code W;

The private key consists of U, V, and data allowing us to map decoding problems into U and V;

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The private key consists of U, V, and data allowing us to map decoding problems into U and V; The parameters are the following:

Parameters	λ	q	n	w	$k = k_U + k_V$	d
Supertubos	128	3	8492	7980	$5605 = 3558 + 2047$	81

Wave Code-based

Wave signature

Wave works in $\mathbb{F}_3$. So, the arithmetic is not "boolean" any more. Also, can someone name a hash function that works in $\mathbb{F}_3$?

[5] Troika: a ternary cryptographic hash function

Wave Code-based

Wave signature

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[5] Troika: a ternary cryptographic hash function

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Wave signature in a Nutshell

Key Generation:

- Generate the SK, that is, subspace code of V and U;
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Verification:

- Hash the message m using a ternary hash function $s = H(m)$;
- Check the weight of $wt(e) \stackrel{?}{=} w$, abort if it is different;
- Check if $s \stackrel{?}{=} eK^T$, abort if it is different.

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Wave signature

Pros and cons of Wave signature:

- Wave has the smallest signatures in code-based: 930 b;
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- It has fast verification.

Cons:

- The public key has size around 4 Mb;
- Key generation and signature are slower than others;
  - It needs to compute the entire Gauss elimination: $O(n^3)$ (constant-time version);

Questions

Thank you for your attention.
Questions?
gustavo@cryptme.in